ISC3, Fall 2020 (A22)
Computer works report 003

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1 Exercice 1 and 2 : Solving Systems of Algebraic Equations

Let $(x, y, z) \mapsto \boldsymbol{F}(x, y, z)$ be the mapping defined by

$$\boldsymbol{F}(x, y, z) = \begin{pmatrix} (x-2)^2 - 1 \\ (y - z - 3)^2 \\ (z+1)^2 - 1 \end{pmatrix}$$

Write a `Scilab` function that implements the mapping of $\boldsymbol{F}$ and a `Scilab` function that computes the Jacobian matrix of $\boldsymbol{F}$.

```
//Implementing the mapping of F
function Fout=F(xvec)
    Fout(1) = ((xvec(1)-2)^2)-1
    Fout(2) = (xvec(2) - xvec(3) - 3)^2
    Fout(3) = ((xvec(3) + 1)^2) - 1
endfunction

//Implementing the Jacobian matrix of F
function Jout=FJac(xvec)
    x = xvec(1); y=xvec(2); z=xvec(3);
    Jout = zeros(3,3);
    Jout = [2*(x-2) 0 0; 0 2*y-2*(z-3) 2*z-2*(y-3);0 0 2*(z+1)];
endfunction
```

Then implement the Newton method that solves $\boldsymbol{F}(x) = 0$ with $x^0 = (0, 0, 0)$ as initial guess.

```
// Implement the Newton method that solves F(x) = 0
```

```
xn = [0;0;0]; itmax = 100; tol=1e-8;
for it = 1:itmax
    J = FJac(xn);
    wn = -J \ F(xn);
    xn = xn + wn
    if (norm(wn)<tol) then
        break
    end
end;
disp(xn);
```

This code gives us the following 3-tuple as solution to the equation $\boldsymbol{F}(x) = 0$:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1. \\ 3.1091812 \\ 0. \end{pmatrix}$$

2 Exercice 3 : Solving Systems of Algebraic Equations - Application to a robotic arm

Consider an articulated arm made of two rods of respective length 4 and 3 . The articulated arm is fixed at the origin $O(0,0)$. The midpoint A that binds the two rods has coordinates $A(x,y)$. The free endpoint is denoted by B.

- Find (x,y) such that $B = (3,3)$.
- Find the two angles θ_1 and θ_2 made by the two rods with the horizontal axis. Use the `Scilab fsolve ()` function to answer the 2 questions.

2.1 Finding the points coordinates

The task at hand is to find the coordinates of a point that satisfies a certain number of conditions. The length between $B(x,y)$ and $A(0,0)$ is 3 and the length between $B(x,y)$ and $C(3,3)$ is 4. Let's implement the situation in `Scilab` :

```
//mapping the system
function Fout=F(xvec)
    Fout(1) = -6*x+x^2+9-6*y+y^2;
    Fout(2) = x^2+y^2-16;
endfunction
x0 = [0;0]
xsol = fsolve(x0, F, 1e^-8)
disp(xsol)
```

This gives us the following solution :

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.1702933 \\ 3.9963734 \end{pmatrix}$$

The coordinates of B in order for it to satisfy the conditions are $(0.1702933, 3.9963734)$

2.2 Finding the angles made by the two rods

Now that we can place B in space, we need to determine the 2 angles $(B\hat{A}X)$ and $(A\hat{B}C)$. This can be done by using the alternate angles principle and transforming the rad measure into degrees.

```
theta1= asin(xsol(2)/4)*(180/%pi)
disp(theta1)
theta2=(acos(xsol(2)/4)+ asin((xsol(2)-3)/3))*(180/%pi)
disp(theta2)
```

The angles are the following : $\theta_1 = 87.559991°$ and $\theta_2 = 21.837782°$.

3 Perfect Pentagon

In order to solve the algebraic system of equations, we need to know what makes a pentagon perfect: if its 5 sides are of equal length. With 5 sides and 5 points we have 10 constraints to evaluate. The following code does this :

```
function Y = F(X)
//declare the vars
    x1 = X(1); y1 = X(2);
    x2 = X(3); y2 = X(4);
    x3 = X(5); y3 = X(6);
    x4 = X(7); y4 = X(8);
    x5 = X(9); y5 = X(10);
    Y = zeros(10,1);//initialize the constraint vector (mapping of the system)

    Y(1) = x1^2+y1^2 - R2; Y(2) = x2^2+y2^2 - R2;
    Y(3) = x3^2+y3^2 - R2; Y(4) = x4^2+y4^2 - R2;
    Y(5) = x5^2+y5^2 - R2;

//declaring the lengths (distance between two points).
    d12 = (x2-x1)^2 + (y2-y1)^2;
    d23 = (x3-x1)^2 + (y3-y1)^2;
```

```
    d34 = (x4-x1)^2 + (y4-y1)^2;
    d45 = (x3-x2)^2 + (y3-y2)^2;
    d51 = (x4-x3)^2 + (y4-y3)^2;
//contraints for the lengths (subscriptions are equal to 0)
    Y(6) = d23 - d12; Y(7) = d34 - d12;
    Y(8) = d45 - d12; Y(9) = d51 - d12;
    Y(10) = y1;
endfunction
```

The other part of the problem is the initial conditioning. This system is highly sensitive to its initial conditions. Only a decent initialization will make the system converge towards the perfect solution.

```
theta = %pi*(1/10);//72°/4
R2 = 4 //length of the sides
X1S = R2*[0;1];
X2S = R2*[cos((9*theta));sin((9*theta))];
X3S = R2*[cos((13*theta));sin((13*theta))];
X4S = R2*[cos((17*theta));sin((17*theta))];
X5S = R2*[cos(theta);sin(theta)];
X0 = [X1S;X2S;X3S;X4S;X5S];

//disp(X0)
[Xsol, val, info] = fsolve(X0, F, 1d-8);
disp(Xsol)
Xmat = matrix(Xsol,2,5); // Reshape the result
scatter(Xmat(1,:),Xmat(2,:));
```

With this initialization we are able to plot the following shape :

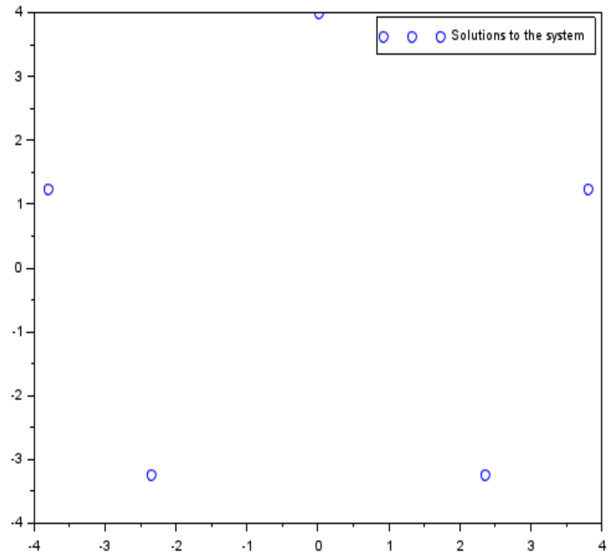

Figure 1: Perfect Pentagon : the points are the solution to the system and verify all the conditions.

4 Intersections of spheres

The objective of this exercise is to determine the intersection(s) of the 3D sphere of center O and radius 1, the sphere of center $(1,0,0)$ and radius 1 and the plane of equation $x+y+z=0$. Write the system of equations to solve, write the corresponding function $F(x)$, compute its Jacobian matrix, and apply the Scilab function fsolve() with the F function and its Jacobian as entry arguments.

The problem is solved by running this script:

```
function Fout = F(xvec)
    Fout(1) = (xvec(1)-1)^2+xvec(2)^2+xvec(3)^2-1;
    Fout(2) = xvec(1)^2+xvec(2)^2+xvec(3)^2-1;
    Fout(3) = xvec(1)+xvec(2)+xvec(3);
endfunction
function J = Fjac(xvec)
    J = zeros(3,3);
    J = [2*(xvec(1)-1) 2*xvec(2) 2*xvec(3);2*xvec(1) 2*xvec(2) 2*xvec(2);1 1 1]
endfunction
```

```
xn = F(xvec); itmax = 200; tol=1e-8;
for it = 1:itmax
    J = Fjac(xn);
    wn = -J \ F(xn);
    xn = xn + wn
    if (norm(wn)<tol) then
        break
    end;
end;

xvec = [0,0,0]
scatter3d(xn(1),xn(2),xn(3));
```

After running the script we get the following coordinates :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.5000000 \\ -0.8090170 \\ 0.3090170 \end{pmatrix}$$

And the following plot :

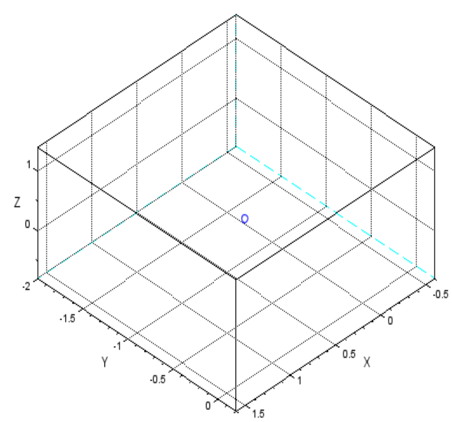

Figure 2: The blue point is the intersection point between the two spheres.